Models of Set Theory II - Winter 2017/2018

Prof. Dr. Peter Koepke, Ana Njegomir Problem sheet 7

Problem 1 (6 points)**.** Let $\langle\langle\mathbb{P}_\alpha, \leq_\alpha, \mathbb{1}_\alpha\rangle \mid \alpha \leq \kappa\rangle$ denote the finite support iteration of the sequence $\langle\langle\dot{\mathbb{Q}}_\alpha, \dot{\leq}_\alpha\rangle \mid \alpha < \kappa\rangle$. Let G be M-generic for $\mathbb{P}_\kappa$ and G_α, H_α be the derived generic filters for $\mathbb{P}_\alpha$ resp. $\dot{\mathbb{Q}}_\alpha^{G_\alpha}$. Show that for each $\alpha \leq \kappa$, $M[G_\alpha] = M[\langle H_\beta \mid \beta < \alpha\rangle]$, where $M[\langle H_\beta \mid \beta < \alpha\rangle]$ is the smallest model N of ZFC with $M \cup \{\langle H_\beta \mid \beta < \alpha\rangle\} \subseteq N$.

Definition. A forcing notion $\mathbb{P}$ is κ-distributive if the intersection of κ open dense sets is open dense, where $D \subset \mathbb{P}$ is open dense if it is dense and if $p \in D$ and $q \leq p$ imply $q \in D$.

Problem 2 (4 points)**.** Suppose that κ is an infinite cardinal and assume that $\mathbb{P}$ is κ-distributive. Let G be $\mathbb{P}$ generic over M. Prove that if $f \in M[G]$ is a function from κ to M, then $f \in M$.

Problem 3 (4 points)**.** Let $\mathbb{T}$ be a normal Suslin tree. Prove that $\mathbb{P}_\mathbb{T}$ is $\aleph_0$-distributive and that it satisfies the countable chain condition, where $(\mathbb{P}_\mathbb{T}, <) = (\mathbb{T}, >)$.

Problem 4 (6 points)**.** Let $\mathbb{P}$ be κ-distributive and $\mathbb{1}_\mathbb{P} \Vdash_\mathbb{P}^M$ "$\dot{\mathbb{Q}}$ is $\check{\kappa}$-distributive". Prove that $\mathbb{P} * \dot{\mathbb{Q}}$ is κ-distributive.

Please hand in your solutions on Monday, November 27 before the lecture.